Early Predictors of High School Mathematics Achievement

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Summary: Elementary school students' knowledge of fractions and division uniquely predicts their high school mathematics achievement, even after controlling for a wide range of relevant variables, suggesting that efforts to improve mathematics education should focus on improving students' learning in those areas.

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Abstract

Improving theories of mathematical development, as well as mathematics education, requires identifying the types of content knowledge that are most predictive of students' long-term learning. To identify these types of knowledge, we examined both concurrent and long-term predictors of high school students' knowledge of algebra and overall mathematics achievement. Analyses of large, nationally-representative, longitudinal data sets from the U.S. and the U.K. revealed that elementary school students' knowledge of fractions and division uniquely predict those students' knowledge of algebra and overall mathematics achievement in high school, five or six years later, even after statistically controlling for other types of mathematical knowledge, general intellectual ability, working memory, and family income and education. Implications of these findings for understanding and improving mathematics learning are discussed.

Early Predictors of High School Mathematics Achievement

Knowledge of mathematics is crucial to educational and financial success in contemporary society and is becoming ever more so. High school students' mathematics achievement predicts college matriculation and graduation, early-career earnings, and earnings growth (Murname, Willett, & Levy, 1995; National Mathematics Advisory Panel (NMAP), 2008). The strength of these relations appears to have increased in recent decades, probably due to a growing percentage of well-paying jobs requiring considerable mathematical proficiency (Murname, et al., 1995). However, many students lack even the basic mathematics competence needed to succeed in typical jobs in a modern economy. Children from low-income and minority backgrounds are particularly at risk for poor mathematics achievement (Hanushek & Rivkin, 2006).

Marked individual and social class differences in mathematical knowledge are present even in preschool and kindergarten (Case & Okamoto, 1996; Starkey, Klein, & Wakeley, 2004) These differences are stable at least from kindergarten through fifth grade; children who start ahead in mathematics generally stay ahead, and children who start behind generally stay behind. ((Duncan et al., 2007; Stevenson & Newman, 1986). Substantial correlations between early and later knowledge are also present in other academic subjects, but differences among children in mathematics knowledge are even more stable than in reading and other areas (Case, Griffin, & Kelly, 1999; Duncan, et al., 2007).

These findings suggest a new type of research that can contribute both to theoretical understanding of mathematical cognition and development and to improving mathematics education. If we can identify specific areas of mathematics that are most consistently predictive of later mathematics proficiency, after controlling for other types of mathematical knowledge, general intellectual ability, and family background variables, we can then determine why those types of knowledge are uniquely predictive and can increase efforts to improve instruction and learning in those areas. The educational payoff is likely strongest in the case of areas that both strongly predict later achievement and in which many children's understanding is poor.

In the present study, we examine sources of continuity in mathematical knowledge from fifth grade through high school. We were particularly interested in testing the hypothesis that early knowledge of fractions is uniquely predictive of later knowledge of algebra and overall mathematics achievement.

One source of this hypothesis was Siegler, Thompson, and Schneider's (2011) integrated theory of mathematical development. The theory proposes that numerical development is a process of progressively broadening the class of numbers that are understood to possess magnitudes and of learning the functions that connect those numbers to their magnitudes. In other words, numerical development involves coming to understand that all real numbers have magnitudes that can be ordered and assigned specific locations on number lines and that can be combined arithmetically. This idea resembles Case and Okamoto's (1996) proposal that the central conceptual structure for whole numbers, a mental number line, is eventually extended to other types of numbers, including rational numbers and negatives. The theory also proposes that a complementary, and equally crucial, part of numerical development is learning that many properties that are true of whole numbers –having unique successors, being countable, including finite number of entities within any given interval, never decreasing with addition and multiplication, etc.– are not true of numbers in general.

One implication of this theory of numerical development is that acquisition of fractions knowledge is crucial to numerical development, rather than being of secondary importance. For most children, fractions provide the first opportunity to learn that a variety of salient and invariant properties of whole numbers are not true of all numbers (e.g., that multiplication does not necessarily produce answers greater than the multiplicands that produce them). This understanding does not come easily; although children receive repeated instruction on fractions starting in third or fourth grade (National Council of Teachers of Mathematics (NCTM), 2006), even high school and community college students often confuse properties of fractions and whole numbers (Schneider & Siegler, 2010; Vosniadou, Vamvakoussi, & Skopeliti, 2008).

This view of fractions as occupying a central position within mathematical development differs substantially from the large majority of theories in the area, which focus on whole numbers and relegate knowledge of fractions and other types of numbers to secondary status or ignore them altogether. To the extent that such theories address development of understanding of fractions at all, it is usually to document ways in which learning about them is hindered by whole number knowledge (e.g., Gelman & Williams, 1998; Wynn, 1995). Nothing in these theories suggests that early knowledge of fractions would uniquely predict later knowledge of mathematics.

Consider some reasons, however, why elementary school students' knowledge of fractions might be crucial for later mathematics learning, for example of algebra. To the extent that instruction focuses on conceptual understanding of fractions, this focus tends to be in elementary school (Kilpatrick, Swafford, & Findell, 2001). Lacking understanding of fractions, students cannot estimate answers even to simple algebraic equations. For example, students who do not understand fractions will not know that in the equation $1/3X = 2/3Y$, X must be twice as large as Y, or that for the equation $3/4X=6$, the value of X must be somewhat, but not greatly, larger than 6. Students who do not understand fraction magnitudes also would be more likely to use flawed equations, because they could not reject them by reasoning that the answers they yielded were impossible. Consistent with this analysis, accurate estimation of fraction magnitudes is closely related to correct use of fractions arithmetic procedures (Hecht & Vagi, 2010; Siegler, et al., 2011). Thus, we hypothesized that 10-year-olds' knowledge of fractions would predict 16-year-olds' algebra knowledge and overall mathematics achievement, even after statistically controlling for other mathematical knowledge, information processing skills, general intellectual ability, and family income and education.

Method

To identify predictors of high school mathematics proficiency, we examined two nationally representative longitudinal data sets: the British Cohort Study (BCS) (Bynner, Ferri, & Shepherd, 1997) and the Panel Study of Income Dynamics-Child Development Study (PSID-CDS) (Hofferth, Davis-Kean, Davis, & Finkelstein, 1998). Detailed descriptions of the samples and measures used in these studies and the statistical analyses that we applied to the data are included in the online supplementary materials; here, we provide a brief overview.

The BCS sample included 3,677 British children born in a single week of 1970. The children completed the Friendly Maths Test and British Ability Scale as 10-year-olds and the APU Arithmetic Test as 10- and 16-year-olds (in 1980 and 1986). At both ages, the Friendly Maths Test included whole number arithmetic and fractions; at age 16, it also included algebra and probability. The British Ability Scale included measures of children's verbal and non-verbal intellectual ability, vocabulary, and spelling. Parents' provided information about their education and income and their children's gender, age, and number of siblings.

The PSID-CDS included a nationally representative sample of 599 U.S. children who were tested in 1997 as 10- to 12-year-olds and in 2002 as 15- to 17-year-olds. At both ages, they completed parts of the Woodcock-Johnson-Revised (WJ-R), a widely used achievement test. The 10- to 12-year-olds performed the Calculation Subtest, which included whole number arithmetic and fractions; the 15- to 17-year-olds completed the Applied Problems Subtest, which included whole number arithmetic, fractions, algebra, geometry, measurement, and probability. Also obtained at ages 10 to 12 were measures of children's non-mathematical intellectual capabilities (backward digit span and passage comprehension); demographic characteristics (gender, age, and number of siblings); and family background (parental education and income).

Results

Our main hypothesis was that early knowledge of fractions would predict high school mathematics knowledge, above and beyond the effects of general intellectual ability, other types of mathematical knowledge, and family background. The data were consistent with this hypothesis.

In the U.K. data (Table 1, Columns 2 and 4), after effects of all other variables were statistically controlled, fractions knowledge at age 10 was the strongest of the five mathematics predictors of age 16 algebra knowledge and mathematics achievement. A one standard deviation increase in early fractions knowledge was uniquely associated with a .15 standard deviation increase in subsequent algebra knowledge and a .16 standard deviation increase in total math achievement ($p<.001$ for both coefficients). In the U.S. data, after effects of all other variables were statistically controlled, the relations between fractions knowledge at ages 10 and 12 and high school mathematics achievement between ages 15 and 17 were of almost identical strength (Table 2, columns 2 and 4). As documented in the online supplementary materials, in both data sets, the predictive power of increments to fractions knowledge was equally strong for children lower and higher in fractions knowledge.

If fractions knowledge continues to be a direct contributor to mathematics achievement in high school, as opposed to having influenced earlier learning but no longer being directly influential, we would expect strong concurrent relations between high school students' knowledge of fractions and their overall mathematical knowledge. High school students' knowledge of fractions did correlate very strongly with their overall mathematics achievement, in both the U.K., r (3675) = .81, $p < .001$, and in the U.S., r (597) = .87, $p < .001$. High school students' fractions knowledge also was closely related to their knowledge of algebra in both the U.K., r (3675) = .68, $p < .001$, and the U.S., r (597) = .65, $p < .001$. Although algebra is a major part of the high school achievement test and fractions a smaller part, the correlation between high school students' knowledge of fractions and their overall mathematics achievement was stronger than the correlation between their algebra knowledge and their overall mathematics achievement in both the U.K. data, r (3675) = .81 versus .73, $\chi^2 = 66.49$, $p < .001$, and in the U.S. set, r (597) = .87 versus .80, $\chi^2 = 15.03$, $p < .001$.

Early knowledge of whole number division also was consistently related to later mathematics proficiency. Among the five mathematics variables in the elementary school tests, the correlation between early division knowledge and later algebra knowledge and overall mathematics achievement was the second strongest in the U.K. data (Table 1) and the strongest in the U.S. data (Table 2). Concurrent correlations between high school students' knowledge of division and their overall mathematics achievement were also substantial: in the U.K., r (3675) = .59; in the U.S., r (597) = .69, p's < .001. To the best of our knowledge, relations between elementary school children's division knowledge and high school students' mathematics proficiency have not been documented, or even hypothesized, previously.

Regressions like those in Tables 1 and 2 place no constraints on the estimated coefficients. To analyze whether early fractions and division knowledge was more strongly predictive of high school mathematics achievement than was early knowledge of whole number addition, subtraction, and multiplication, we re-estimated our regression models imposing an equality constraint on the coefficients on the coefficients for fractions and division, as well as an equality constraint on the coefficients for addition, subtraction, and multiplication. We then tested whether the two sets of pooled coefficients differed from each other. The predictive relation was stronger for fractions and division in both the U.K. data (F (1, 3664) =28.79, $p <$ .001) and the U.S. data (F (1, 558) = 9.72, $p < .005$).

The greater predictive power of fractions and division was not due to their generally predicting intellectual outcomes more accurately. When the models shown in columns (2) and (4) in Table 1 and 2 were applied to predicting high school students' literacy (spelling and vocabulary on the BCS, passage comprehension and working memory on the PSID-CDS), estimated coefficients on early fractions and division knowledge did not differ from those on addition, subtraction, and multiplication knowledge on either measure in the U.S. data and only differed for vocabulary in the U.K. data, F (1, 3675) = 5.53, $p < .05$, Appendix S4).

Discussion

The present findings demonstrate that elementary school students' knowledge of fractions and division predicts their mathematics achievement in high school, above and beyond the contributions of whole number arithmetic knowledge, verbal and non-verbal IQ, working memory, and family education and income. The relations of fractions and division to math achievement were stronger than for addition, subtraction, multiplication, verbal IQ, and parental education and income. These results were consistent across data sets from the U.K. and the U.S.

The similarity in the U.S. and U.K. samples of the strength of the relations of different predictor variables to algebra and overall mathematics achievement data, despite differences in the demographic characteristics of the samples, the specifics of the tests, and the times at which the data were obtained, is one reason for confidence in the generality of the findings.

The correlation between elementary school fractions knowledge and high school algebra and mathematics achievement was expected, but the relation between early division knowledge and later mathematical knowledge was not. Fractions and division are inherently related – N/M means N divided by M -- but the finding that early knowledge of fractions and division accounted for independent variance in later algebra knowledge and overall mathematics achievement indicated that neither relation explained the other.

Several other processes seem likely to have contributed to the relation. Mastery of whole number division, like mastery of fractions, is required to solve many algebra problems (for example, to apply the quadratic equation). Also as with fractions, high percentages of students fail to master division; thus, presented a seemingly easy PSID-CDS problem in which a boy wants to fly on a plane that travels four hundred miles per hour to visit his grandmother who lives fourteen hundred air miles away, only 56% of high school students correctly indicated how long the flight would take. More speculatively, the poor knowledge of fraction magnitudes documented in previous studies might lead to students giving up trying to make sense of mathematics, and thus rely on rote memorization in subsequent mathematics learning.

An alternative interpretation is that the unique predictive value of fractions and division stems from those operations being more difficult than addition, subtraction, and multiplication, and thus measuring more advanced thinking. Two types of data argue against this interpretation, however. One is that fractions and division were not uniquely predictive of subsequent reading related skills, as should have been the case if their predictive value was due solely to their greater difficulty. The other is that the Spline tests showed that the predictive relation of fractions and division was just as strong for students with greater as lesser mathematics achievement. Thus, the unique predictive value of early fractions and division knowledge seems to be due to the combination of many students not mastering fractions and division and to those operations being essential for more advanced mathematics, rather than to the relatively great difficulty of fractions and division per se.

Over 30 years of nationwide standardized testing of mathematics knowledge, scores of U.S. high school students have barely budged (National Mathematics Advisory Panel (NMAP), 2008). The present findings imply that mastery of fractions and division is needed if substantial improvements in algebra and other aspects of high school mathematics are to be achieved. One likely source of students' limited mastery of fractions and division is that many U.S. teachers lack a firm conceptual understanding of both fractions and division. For example, in several studies, the majority of U.S. elementary and middle school teachers have been unable to generate even a single explanation for why the invert and multiply algorithm is a legitimate way to solve fractions division problems; in contrast, most teachers in Japan and China generated two or three explanations in response to the same question (Ma, 1999; Moseley, Okamoto, & Ishida, 2007). These and the present results suggest that improved teaching of fractions and division could yield substantial improvements in students' learning, not only of fractions and division but of more advanced mathematics as well.

Bibliography

Bynner, J., Ferri, E., & Shepherd, P. (Eds.). (1997). *Twenty-something in the 1990s: getting on, getting by, getting nowhere*. Aldershot: Ashgate.

Case, R., Griffin, S., & Kelly, W. M. (1999). Socioeconomic gradients in mathematical ability and their responsiveness to intervention during early childhood. In D. P. Keating & C. Hertzman (Eds.), *Developmental health and the wealth of nations: Social, biological, and education dynamics* (pp. 125-152). New York: Guilford Press.

Case, R., & Okamoto, Y. (1996). The role of central conceptual structures in the development of children's thought. *Monographs of the Society for Research in Child Development, 61(Nos. 1-2)*.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Pagani, L., . . . Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*, 1428-1446.

Gelman, R., & Williams, E. (1998). Enabling constraints for cognitive development and learning: Domain specificity and epigenesis. In W. Damon, D. Kuhn & R. S. Siegler (Eds.), *Handbook of child psychology* (5th ed., Vol. 2: Cognition, perception & language). New York: Wiley.

Hanushek, E. A., & Rivkin, S. G. (2006). School quality and the black-white achievement gap: NBER Working Papers, 12651. National Bureau of Economic Research, Inc. .

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology, 102*(4), 843-858.

Hofferth, S., Davis-Kean, P. E., Davis, J., & Finkelstein, J. (1998). The Child Development Supplement of the Panel Study of Income Dynamics, 1997 User Guide *available at* http://www.nber.org/~kling/surveys/PSID.html. Ann Arbor, MI: The University of Michigan, Survey Research Center Institute for Social Research.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press (National Research Council).

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Moseley, B. J., Okamoto, Y., & Ishida, J. (2007). Comparing U.S. and Japanese elementary school teachers' facility for linking rational number representations. *International Journal of Science and Mathematics Education, 5*, 165-185.

Murname, R. J., Willett, J. B., & Levy, F. (1995). The growing importance of cognitive skills in wage determination. *Review of Economics and Statistics, 78*, 251-266.

National Council of Teachers of Mathematics (NCTM). (2006). Curriculum focal points for prekindergarten through grade 8 mathematics. Washington, DC: National Council of Teachers of Mathematics. Pdf available at http://www.nctm.org/focalpoints/down-loads.asp.

National Mathematics Advisory Panel (NMAP). (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC, U.S. Department of Education.

Schneider, M., & Siegler, R. S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Perception and Performance, 36*, 1227-1238.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Development, 62*, 273-296.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly, 19*, 99-120.

Stevenson, H. W., & Newman, R. S. (1986). Long-term prediction of achievement and attitudes in mathematics and reading. *Child Development, 57*, 646-659.

Vosniadou, S., Vamvakoussi, X., & Skopeliti, I. (2008). The framework theory approach to conceptual change. In S. Vosniadou (Ed.), *International handbook of research on conceptual change* (pp. 3-34). Mahwah, NJ: Erlbaum.

Wynn, K. (1995). Infants possess a system of numerical knowledge. *Current Directions in Psychological Science, 4*, 172-177.

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The data reported in the article are archived and available at the following websites: The British Cohort Study: http://www.cls.ioe.ac.uk/ and the Panel Study of Income Dynamics-Child Development Supplement: http://psidonline.isr.umich.edu/

	Age 16 Algebra				Age 16 Total math score			
Model:	(1)		(2)		(3)		(4)	
	Bivariate		Multiple regression		Bivariate		Multiple regression	
Age 10 domains of math:								
Fractions	.42	***	.15	***	.46	***	.16	***
	(.02)		(.02)		(.02)		(.02)	
Addition	.20	***	.00		.26	***	.05	**
	(.02)		(.02)		(.02)		(.02)	
Subtraction	.22	***	.04	*	.24	***	.03	
	(.02)		(.02)		(.02)		(.02)	
Multiplication	.32	***	.06	***	.37	***	.08	***
	(.02)		(.02)		(.02)		(.02)	
Division	.37	***	.13	***	.40	***	.12	***
	(.02)		(.02)		(.02)		(.02)	
Age 10 ability:								
Verbal IQ	.39	***	.11	***	.42	***	.10	***
	(.02)		(.02)		(.02)		(.02)	
Non-verbal IQ	.41	***	.17	***	.46	***	.19	***
	(.02)		(.02)		(.02)		(.02)	
Child characteristics:								
Girl	-.02		.00		-.01		.00	
	(.02)		(.02)		(.02)		(.01)	
Child's age	.01		-.03	*	.01		-.03	*
	(.02)		(.02)		(.02)		(.01)	
Background characteristics:								
Log Mean household income	.38	***	.08	*	.40	***	.09	*
	(.04)		(.03)		(.04)		(.04)	
Parent education	.27	***	.10	***	.29	***	.10	***
	(.02)		(.02)		(.02)		(.02)	
Number of siblings	-.05	**	-.01		-.09		-.05	***
	(.02)		(.01)		(.02)		(.01)	
N	3,677		3,677		3,677		3,677	
Mean R^2 for Model 2			.29				.35	

Table 1: Early predictors of high school mathematics achievement: BCS data. Standardized coefficients and standard errors from bivariate and multiple regression models of age 16 math assessments on age 10 math skills and child and family characteristics, British Cohort Study data.
Table notes: *p<.05, ** p<.01, *** p<.001.

Models 1 and 3 show bivariate associations; Models 2 and 4 show results a single multiple regression that includes all predictors.
All independent variables, child and background characteristics are standardized.
Parameter estimates and standard errors are based on 20 multiply-imputed data sets.

	Age 15-17 Algebra				Age 15-17 Total math score			
Model:	(1)		(2)		(3)		(4)	
	Bivariate (*r*)		Multiple regression (*B*)		Bivariate (*r*)		Multiple regression (*B*)	
Age 10-12 domains of math:								
Fractions	.41	***	.17	*	.49	***	.18	**
	(.06)		(.08)		(.05)		(.06)	
Addition	.26	***	.09		.30	***	.05	
	(.06)		(.06)		(.06)		(.05)	
Subtraction	.26	***	.04		.39	***	.12	*
	(.05)		(.05)		(.05)		(.05)	
Multiplication	.31	***	.00		.43	***	.02	
	(.05)		(.06)		(.05)		(.05)	
Division	.40	***	.19	***	.53	***	.26	***
	(.05)		(.06)		(.05)		(.06)	
Age 10-12 ability:								
Digit Span Backward	.29	***	.10		.33	***	.08	
	(.06)		(.06)		(.05)		(.05)	
Passage Comprehension	.38	***	.11		.51	***	.20	***
	(.05)		(.06)		(.05)		(.05)	
Child characteristics:								
Girl	-.06		-.09		-.08		-.13	***
	(.05)		(.05)		(.05)		(.04)	
Child's age (years)	.04		-.18	***	.06		-.22	***
	(.05)		(.05)		(.05)		(.04)	
Background characteristics:								
Log Mean Family Income 94-96	.31	***	.05		.38	***	.12	*
	(.06)		(.06)		(.06)		(.06)	
Parent Education (highest)	.39	***	.19	***	.41	***	.11	
	(.05)		(.05)		(.06)		(.06)	
No. Siblings	-.17	**	-.03		-.18	**	-.03	
	(.06)		(.05)		(.06)		(.04)	
N	599		599		599		599	
Mean R^2 for Model 2			.35				.52	

Table 2: Early predictors of high school mathematics achievement: PSID Data. Standardized coefficients and standard errors from bivariate and multiple regression models of age 15-17 math assessments on age 10-12 math skills and child and family characteristics, Panel Study of Income Dynamics data.
Table notes: *p<.05, ** p<.01, *** p<.001.
Models 1 and 3 show bivariate associations; Models 2 and 4 show results a single multiple regression that includes all predictors.
All independent variables, child and background characteristics are standardized.
Parameter estimates and standard errors are based on 20 multiply-imputed data sets.

Supporting Online Material for

Early Predictors of High School Mathematics Achievement

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This part contains the following sections:

Materials and Methods
Tables S1 to S5 (in their own files)

Materials and Methods

Data

The present data come from two nationally representative, longitudinal datasets containing detailed mathematics assessments at two different time points: the U.K. 1970 British Cohort Study (BCS) the U.S. Child Development Supplement of the Panel Study of Income Dynamics (PSID-CDS).

British Cohort Study (BCS). The 1970 British Cohort Study is a longitudinal study following into adulthood all individuals born in Great Britain during a single week in April, 1970 (Bynner, Ferri, & Shepherd, 1997). Data collection sweeps for BCS have taken place when the cohort members were aged 5, 10, 16, 26, 30, 34, and most recently 38 years (J. Elliott & Shepherd, 2006). The birth sample of 17,196 infants was approximately 97% of the target birth population. The responding sample at age 10 was 14,350 (83%) and at age 16 was 11,206 (65%). A teachers' strike at the same time as the age 16 sweep reduced the number of cohort members for which we have achievement data: math test scores are only available for 21% of the cohort for that age group. The current sample is therefore made up of the 3,677 individuals (52% male) whose mathematics knowledge was assessed at ages 10 and 16.

Analyses of response bias in these data have shown that the achieved samples did not differ from their target samples across a number of critical variables (social class, parental education, and gender), despite a slight under-representation of the most disadvantaged groups (Plewis, Calderwood, Hawkes, & Nathan, 2004). Bias due to attrition of the sample during childhood has also been shown to be minimal (Plewis, et al., 2004). Other analyses using these data find that the mathematics test score available for the reduced age 16 sample is as good a predictor of subsequent labor market outcomes as a more general achievement measure available for the whole age 16 cohort, further increasing our confidence that the lower response rate did not adversely affect our results (Maurin & McNally, 2009). Finally, these same analyses also demonstrated high comparability in the distribution and predictive power of mathematics test scores with another U.K. longitudinal birth cohort, the 1958 National Child Development Survey, which did not suffer the same attrition problem.

Data for the BCS were collected from a variety of sources, including the mother, health care professionals, teachers, school health service personnel, and the individual child, and in a number of ways, including paper and electronic questionnaires, clinical records, medical examinations, tests of ability, educational assessments, and diaries. Data and documentation are available at http://www.cls.ioe.ac.uk/

Panel Study of Income Dynamics – Child Development Study (PSID-CDS). The Panel Study of Income Dynamics began in 1968 by drawing a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. Information on these individuals and their descendants has been collected continuously (annually through 1993, biennially since then). The information includes data on employment, income, wealth, expenditures, health, marriage, childbearing, child development, philanthropy, education, and numerous other topics.

In 1997, all PSID families who had children between birth and 12 years of age were recruited to participate in the Child Development Supplement of the PSID (Hofferth, Davis-Kean, Davis, & Finkelstein, 1998), which provides the data used in this article. The CDS includes up to two children selected randomly from each PSID family that agreed to participate (CDS-I). The CDS-I collected data on 2,394 families (88% of eligible families) and their 3,563 children. The families who remained active in the PSID were reassessed in 2002 and 2003 (CDS-II). The CDS-II collected data on the 2091 families (91% of those in the CD-I) and their 2,907 children.

Like the BCS, the CDS has conducted interviews, assessments, and home observations that provide information on a broad range of developmental outcomes in the areas of health, psychological well being, social and cognitive development, and education, as well as a range of measures of the family neighborhood and school environment, among other variables. Data and documentation are available at http://psidonline.isr.umich.edu/

For the present study, 2,523 children were included who, in the CDS-II, had complete data in the three subtests of the Woodcock Johnson Revised Tests of Achievement (WJ-R): Applied Problems (Word Math Problems), Letter-Word Identification (Vocabulary), and Passage Comprehension (Reading Comprehension). The sample was further reduced to 599 when children were selected for the 10 to 12 years target age range for our study.

Analysis procedures

Our analyses of data from the BCS and PSID-CDS use multiple regression to assess the predictive importance of different domains of math measured at age 10 (in the BCS) or ages 10-12 (for the PSID-CDS) for advanced mathematic achievement at age 16 (in the BCS) or ages 15-17 (in the PSID-CDS). In both datasets, baseline achievement tests were used to form our key measures of knowledge of fractions, addition, subtraction, multiplication and division. Each of these five subscales was measured by the proportion of items correct on that subscale. To facilitate comparisons across the subscales, all five are standardized to mean zero and unit standard deviations.

To avoid attributing to these math components predictive power that is more properly attributed to general cognitive ability or family background, both sets of analysis control for measures of the child's intellectual ability, age, parents' social class and highest level of education/maternal literacy, as well as family income and family size. As explained below, we also conducted falsification tests using age 16 (in the BCS) or 15-17 (in the PSID) literacy achievement measures as dependent variables. To account for missing data in both data sets we used multiple imputations by chained equations (ICE) as implemented in STATA (Royston, 2005). In both cases, we use the multiple imputation procedures to produce 20 data sets, including all the variables in the analysis model for both data sets. For the BCS only, the addition of fathers' social class, father absence, birth weight, and an age 5 copying task were used to assist in the imputation calculations. In addition, the PSID-CDS data are weighted using weights supplied by the study to account for differential sampling fractions and case attrition.

Measures

In the BCS, age 10 mathematics achievement was measured by the "Friendly Maths Test" developed specifically for the BCS in collaboration with specialists in primary mathematics (Social Statistics Research Unit, 1982). The test consists of 72 multiple choice questions and assesses knowledge of the rules of arithmetic, number skills, and fractions. The reliability of the test is α=.93. For the purposes of this study, the math score was divided into items measuring whole number addition, subtraction, multiplication, division and fraction.

Sixteen year olds were given a timed arithmetic test consisting of 60, increasingly difficult, problems. The test began with basic arithmetic expressions and simple word problems and moved on to questions on fractions, percentages, algebra, estimation of area and probability. Literacy at age 16 was assessed by separate tests in advanced spelling and vocabulary.

Children's ability was measured at age 10 using the British Ability Scales (C. D. Elliott, Murray, & Pearson, 1978), a cognitive test battery for children between 3 and 17 years of age that assesses verbal and non-verbal reasoning. Verbal sub-scales comprised word definitions (37 items, α=.83) and word similarities (42 items, α=.80). Non-verbal sub-scales comprised recall of digits (34 items, α=.84) and matrices (28 items, α=.87).

Additional child level characteristics included in our analysis include child gender (0 = male, 1 = female) and age, in years, at the time of testing. Given the logistical complications associated with the size of the BCS cohort and the difficulties of administering the age 10 tests in schools, children's ages ranged from 9.3 to 11.4 years, with an average of 10.2 years.

Highest household education is measured in terms of parents' estimated years of schooling, ranging from 10 to 17 years, with an average of 12.5. This measure has the advantage of including vocational as well as academic qualifications and indicates that less than a third (29%) of this cohort's parents went on to post-compulsory education. Gross weekly family income, before deductions, was measured in bands when children were age 10. Our analysis uses the natural logarithm of the midpoint of each band. Baseline information on family size was provided by the mother at the time of study enrolment, i.e. birth, and was updated with further detail provided at the age 10 assessments.

Missing data on these variables was multiply imputed based on 20 data imputations. All variables were standardized to z-scores. Descriptive statistics for all variables used in our analyses are shown in Table S1. Correlations among age 10 and 16 math and literacy measures in the BCS are provided in Table S2.

In the PSID-CDS, 10 to 12 year-old children were assessed using the Calculation subtest of the *Woodcock-Johnson Psycho-Educational Battery – Revised* (WJ-R) in CDS- I and the Applied Problems, Passage Comprehension, and Letter-Word Identification subtests of the WJ-R in CDS-II (at 15-17 years of age) (Woodcock & Johnson, 1989/1990). The Woodcock-Johnson has been widely used in national longitudinal studies, and has good psychometric properties. The split-half reliabilities reported for the group of 10-17 year-old children ranges between .78 and .94 (Woodcock & Johnson, 1989/1990). For the purpose of this study, the Calculation and Applied Problems subscales were divided into the subcomponents of specific mathematics skill areas — addition, subtraction, multiplication, division, fractions, and algebra. These specific mathematics skills were measured in proportion of correct responses. The child's total mathematics score was measured as the total score of Applied Problems at 15-17 years old. Literacy at ages 15-17 years of age were assessed by separate tests of vocabulary (WJ-Letter Word) and reading comprehension (WJ-Passage Comprehension).

Children's general intellectual ability was measured in the PSID by their short-term memory scores on the Digit Span subtest of the WISC-R (Wechsler, 1974). Children were given a series of numbers and asked to repeat them either forward or backward. The proportion correct for the 14 items of the backward digit span measure at Wave I (CDS-I) was used.

As in the BCS, additional child level characteristics (gender and age) were included in the analysis. For gender, males were assigned a code of 0 and females a code of 1. Age was measured at the time of children's assessment (CDS-I) and, it is expressed as the number of years at the time of the interview. The average age at CDS-I was 11.5 years of age and range between 10.0 and 12.9 years of age.

The educational level of the family was measured by the highest education of the head of household and his/her spouse. The values range from 0-17, with a mean of 13.24, which indicates a mean of slightly more than a high school education for the sample. Family income was measured using the natural logarithm of average income of the family from 1994, 1995 and 1996. The number of siblings represents the number of children living in the house at the time of the 1997 interview.

All variables were standardized to z-scores using weighted mean and standard deviation for the 10-12 year olds in the CDS-I from the imputed data. Because the CDS is intended to be a nationally representative sample of the children and their primary caregivers in the U.S, sample weights were used to account for differential probabilities of selection. The CDS weights also adjust for attrition across interviewing waves. The child-level weight was used in all of the analyses presented in this paper (Panel Study of Income Dynamics Child Development Supplement).

Descriptive statistics for all variables in our analyses are shown in Table S1. Correlations among age 10 and 16 mathematics and literacy measures in the PSID-CDS are provided in Table S3.

Falsification tests

One possible concern regarding the predictive power of fractions and division knowledge that is demonstrated in the paper is that the findings might represent some kind of method effect. For example, these two subscales might best measure a student's general performance on standardized tests. To allay these concerns, we conducted the falsification tests presented in Tables S4 and S5. The regression models are identical to those used in the article, with the exception that the dependent variables are measures of reading and spelling rather than math achievement. In only one of the four cases is the coefficient on age 10 (in the BCS) and ages 10-12 (in the PSID) fractions statistically significant at conventional levels.

Nonlinear Effects

To explore the generality of these relations over achievement levels, we fit piecewise linear (spline) functions that allowed for different slopes for children in the bottom and top halves of the baseline distributions of fractions and division knowledge. Although slope estimates for fractions knowledge were somewhat steeper for children in the top as opposed to bottom half of the fractions knowledge distributions, in only one of the four cases was the slope difference statistically significant at $p<.05$. There were no clear patterns for slope differences in division knowledge. Overall, the spline analyses showed that the predictive power of early fractions and division knowledge to later algebra and overall mathematics achievement was similar for children with low and high levels of fractions and division knowledge.

Bibliography

Bynner, J., Ferri, E., & Shepherd, P. (Eds.). (1997). *Twenty-something in the 1990s: getting on, getting by, getting nowhere*. Aldershot: Ashgate.

Elliott, C. D., Murray, D. J., & Pearson, L. S. (1978). *British Ability Scales*. Slough: National Foundation for Educational Research.

Elliott, J., & Shepherd, P. (2006). Cohort profile: 1970 British Birth Cohort (BCS70). *International Journal of Epidemiology, 35*, 836-843.

Hofferth, S., Davis-Kean, P. E., Davis, J., & Finkelstein, J. (1998). The Child Development Supplement of the Panel Study of Income Dynamics, 1997 User Guide *available at* http://www.nber.org/~kling/surveys/PSID.html. Ann Arbor, MI: The University of Michigan, Survey Research Center Institute for Social Research.

Maurin, E., & McNally, S. (2009). *The consequences of ability tracking for future outcomes and social mobility (Mimeo)*. London: London School of Economics.

Panel Study of Income Dynamics Child Development Supplement. *User Guide for CDS-II*. Retrieved May 28, 2010, from http://psidonline.isr.umich.edu/CDS/cdsii_userGd.pdf.

Plewis, I., Calderwood, L., Hawkes, D., & Nathan, G. (2004). *National Child Development Study and 1970 British Cohort Study technical report: Changes in the NCDS and BCS70 populations and samples over time*. London: University of London, Institute of Education, Centre for Longitudinal Studies.

Royston, P. (2005). Multiple imputation of missing values: Update of ice. *The Strate Journal, 5*(4), 527-537.

Social Statistics Research Unit. (1982). 1970 British Cohort Study Ten-year follow-up: Guide to data available at the ESRC Data Archive. London: City University.

Wechsler, D. (1974). *Manual for the Wechsler Intelligence Scale for Children—Revised*. New York: Psychological Corporation.

Woodcock, R. W., & Johnson, M. B. (1989/1990). *Woodcock-Johnson Psycho-educational Battery—Revised.* . Allen, TX: DLM Teaching Resources.

Table S1: Descriptive characteristics of the study samples

British Cohort Study	***Mean***	***Std. Dev.***	***Min***	***Max***
Age 16 mathematics domain:				
Fractions	.72	(.23)	0	1
Division	.89	(.18)	0	1
Algebra	.55	(.27)	0	1
Overall Math score	.64	(.21)	0	1
Age 16 literacy domains:				
Spelling	.78	(.21)	0	1
Vocabulary	.55	(.16)	.01	1
Age 10 mathematics domain:				
Fractions	.62	(.27)	0	1
Addition	.95	(.10)	0	1
Subtraction	.91	(.13)	.2	1
Multiplication	.78	(.23)	0	1
Division	.72	(.27)	0	1
Age 10 ability:				
Verbal IQ	100.00	(15.00)	46.27	151.56
Non-verbal IQ	100.00	(15.00)	46.34	158.13
Child characteristics:				
Girl	.54	(.50)	0	1
Child's age (years)	10.15	(.21)	9.40	11.37
Background characteristics:				
Log Family Income	4.77	(.51)	2.86	5.70
Highest household education (number of years)	12.48	(2.50)	10	17
No. Siblings	1.42	(1.00)	0	8

Table S1: Descriptive characteristics of the study samples (continued)

Panel Study of Income Dynamics	***Mean***	***Std. Dev.***	***Min***	***Max***
Age 15-17 domains of math:				
Fractions	.58	(.28)	0	1
Division	.76	(.33)	0	1
Algebra	.36	(.25)	0	1
Total Math score	.73	(.11)	.37	1
Age 15-17 literacy domains:				
Letter Words	.89	(.08)	.44	1
Passage Comprehension	.72	(.10)	.28	1
Age 10-12 domains of math:				
Fractions	.20	(.21)	0	1
Addition	.85	(.10)	.38	1
Subtraction	.93	(.12)	0	1
Multiplication	.74	(.19)	0	1
Division	.65	(.32)	0	1
Age 10-12 ability:				
Digit Span Backward	5.60	(2.05)	0	14
Passage Comprehension	.60	(.11)	0	0.95
Child characteristics:				
Girl	.52	(.50)	0	1
Child's age (years)	11.50	(.89)	10.00	12.99
Background characteristics:				
Log Mean Family Income 94-96	10.60	(.80)	7.48	13.63
Parent Education (highest)	13.24	(3.28)	0	17
No. Siblings	1.63	(1.14)	0	7

Table notes: BCS results are based on 20 multiple imputations (N=3,677 each). PSID results based on 20 multiple imputations (N=599 each) and weighted by 2002 child level weights.

Table S2: Correlations among mathematics, literacy and cognitive ability variables, British Cohort Study

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Age 16 domains of math:													
(1)	Fractions	1												
(2)	Division	.52	1											
(3)	Algebra	.68	.41	1										
(4)	Total Math score	.81	.59	.73	1									
	Age 16 literacy domains:													
(5)	Spelling	.29	.24	.27	.35	1								
(6)	Vocabulary	.52	.36	.50	.58	.27	1							
	Age 10 domains of math:													
(7)	Fractions	.44	.26	.43	.47	.16	.39	1						
(8)	Addition	.24	.20	.20	.26	.10	.20	.28	1					
(9)	Subtraction	.21	.17	.22	.24	.12	.17	.28	.29	1				
(10)	Multiplication	.34	.25	.32	.37	.17	.27	.43	.28	.32	1			
(11)	Division	.37	.26	.37	.40	.15	.28	.48	.26	.30	.47	1		
	Age 10 ability:													
(12)	Verbal IQ	.40	.24	.39	.43	.19	.49	.53	.26	.23	.34	.35	1	
(13)	Non-verbal IQ	.42	.30	.42	.46	.20	.38	.50	.30	.26	.39	.41	.52	1

Table notes: All correlations significant at $p<.01$. Results are based on 20 multiple imputations (N=3,677 each).

Table S3: Correlations among math, literacy and cognitive ability variables, Panel Study of Income Dynamics

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Age 15-17 domains of math:													
(1)	Fraction	1												
(2)	Division	.59	1											
(3)	Algebra	.65	.48	1										
(4)	Total Math score	.87	.69	.80	1									
	Age 15-17 literacy domains:													
(5)	Letter Words	.50	.49	.43	.60	1								
(6)	Passage Comprehension	.56	.51	.54	.67	.71	1							
	Age 10-12 domains of math:													
(7)	Fractions	.46	.34	.41	.49	.40	.41	1						
(8)	Addition	.25	.25	.26	.30	.22	.19	.37	1					
(9)	Subtraction	.31	.36	.26	.39	.38	.35	.26	.38	1				
(10)	Multiplication	.36	.37	.31	.43	.47	.43	.51	.36	.45	1			
(11)	Division	.48	.41	.40	.53	.49	.44	.58	.33	.40	.64	1		
	Age 10-12 ability:													
(12)	Digit Span Backward	.27	.24	.29	.33	.30	.36	.27	.21	.23	.32	.31	1	
(13)	Passage Comprehension	.47	.42	.38	.51	.65	.64	.44	.32	.41	.45	.46	.35	1

Table notes: All correlations significant at p<.01. Results based on 20 multiple imputations (N=599 each) and weighted by 2002 child level weights.

Table S4: Standardized coefficients and standard errors from regression models of age 16 literacy assessments on age 10 math skills and child and family characteristics, British Cohort Study

	Spelling				Vocabulary			
Model:	(1)		(2)		(3)		(4)	
	Bivariate		Multiple regression		Bivariate		Multiple regression	
Age 10 domains of math:								
Fractions	.16	***	.02		.38	***	.09	***
	(.02)		(.02)		(.02)		(.02)	
Addition	.10	***	.00		.21	***	.03	
	(.02)		(.02)		(.02)		(.02)	
Subtraction	.12	***	.04		.17	***	.00	
	(.02)		(.02)		(.02)		(.02)	
Multiplication	.17	***	.05	*	.27	***	.03	
	(.02)		(.02)		(.02)		(.02)	
Division	.16	***	.03		.28	***	.04	
	(.02)		(.02)		(.02)		(.02)	
Age 10 ability:								
Verbal IQ	.19	***	.09	***	.49	***	.30	***
	(.02)		(.02)		(.02)		(.02)	
Non-verbal IQ	.20	***	.08	***	.38	***	.10	***
	(.02)		(.02)		(.02)		(.02)	
Child characteristics:								
Girl	.14	***	.14	***	.00		.03	
	(.02)		(.02)		(.02)		(.02)	
Child's age	.01		.00		.02		-.02	
	(.02)		(.02)		(.02)		(.02)	
Background characteristics:								
Log family income	.19	***	.05		.39	***	.05	
	(.03)		(.04)		(.03)		(.03)	
Parent education	.13	***	.04	*	.32	***	.14	***
	(.02)		(.02)		(.02)		(.05)	
Number of siblings	-.09	***	-.07	***	-.12	***	-.05	***
	(.02)		(.02)		(.02)		(.02)	
N	3677		3677		3677		3677	
Mean R^2 for model 2			.09				.30	

Table notes: * *p*<.05, ** p<.01, *** p<.001
Model 1 shows bivariate associations; Model 2 shows results a single multiple regression that includes all predictors
All independent variables, child and background characteristics are standardized
Parameter estimates and standard errors are based on 20 multiply-imputed data sets

Table S5: Standardized coefficients and standard errors from regression models of age 16 literacy assessments on age 10 math skills and child and family characteristics, Panel Study of Income Dynamics

	Letter Word				Passage Comprehension			
Model:	(1)		(2)		(3)		(4)	
	Bivariate		Multiple regression		Bivariate		Multiple regression	
Age 10-12 domains of math:								
Fractions	.40	***	.03		.41	***	.04	
	(.05)		(.05)		(.05)		(.06)	
Addition	.22	***	-.05		.19	***	-.06	
	(.05)		(.04)		(.06)		(.05)	
Subtraction	.38	***	.07		.35	***	.05	
	(.06)		(.06)		(.05)		(.05)	
Multiplication	.47	***	.14	*	.43	***	.11	*
	(.06)		(.06)		(.05)		(.05)	
Division	.49	***	.14	*	.44	***	.08	
	(.06)		(.07)		(.05)		(.05)	
Age 10-12 ability:								
Digit Span Backward	.30	***	.03		.36	***	.10	*
	(.05)		(.04)		(.05)		(.04)	
Passage Comprehension	.65	***	.46	***	.65	***	.43	***
	(.06)		(.06)		(.04)		(.05)	
Child characteristics:								
Girl	.15	**	.05		.13	*	.03	
	(.05)		(.04)		(.05)		(.04)	
Child's age (years)	.17	***	-.08		.13	**	-.08	
	(.05)		(.05)		(.05)		(.04)	
Background characteristics:								
Log Mean Family Income 94-96	.33	***	.08		.39	***	.05	
	(.05)		(.05)		(.06)		(.05)	
Parent Education (highest)	.32	***	.05		.47	***	.23	***
	(.07)		(.06)		(.05)		(.05)	
No. Siblings	-.06		.06		-.15	***	.03	
	(.06)		(.04)		(.05)		(.04)	
N	599		599		599		599	
Mean R^2 for model 2			.51				.53	

Table notes: * *p*<.05, ** p<.01, *** p<.001
Model 1 shows bivariate associations; Model 2 shows results a single multiple regression that includes all predictors. All independent variables, child and background characteristics are standardized. Parameter estimates and standard errors are based on 20 multiply-imputed data sets Models are weighted by 2002 child level weights and adjusted for the clustering of children within the same family.